

Unit III: Dynamics

Dynamics is the study of why objects move – it is the study of forces. Kinematics and Dynamics together form what is called Mechanics. Two people were instrumental in the birth of this branch of physics:

Galileo Galilei

- Born in Italy in 1564 and died in 1642
- Remembered for his work in astronomy, mathematics, and physics
- Made the Catholic Church grumpy because he challenged Aristotelian notions about motion by performing experiments
- In 1633 the Inquisition forced him to renounce his theories and placed him under house arrest

Sir Isaac Newton

- Born in England in 1642 and died in 1727
- Invented Calculus
- Came up with three Laws of Motion
- Not friendly, but always acknowledged the work of those before him, especially Galileo:
 - “If I have seen further than other men, it is because I have stood on the shoulders of giants.”

Force

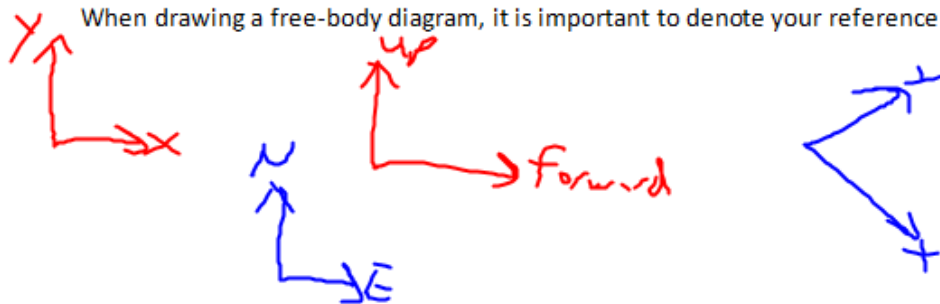
A force is push or a pull. Since these can have different magnitudes and directions, force is a vector quantity. Any force acting on an object can change its shape, its velocity, or both.

The symbol for force is \vec{F} and the unit is the Newton (N). One Newton is equal to 1 kgm/s^2 . Direction is denoted with whatever system is most convenient for the situation.

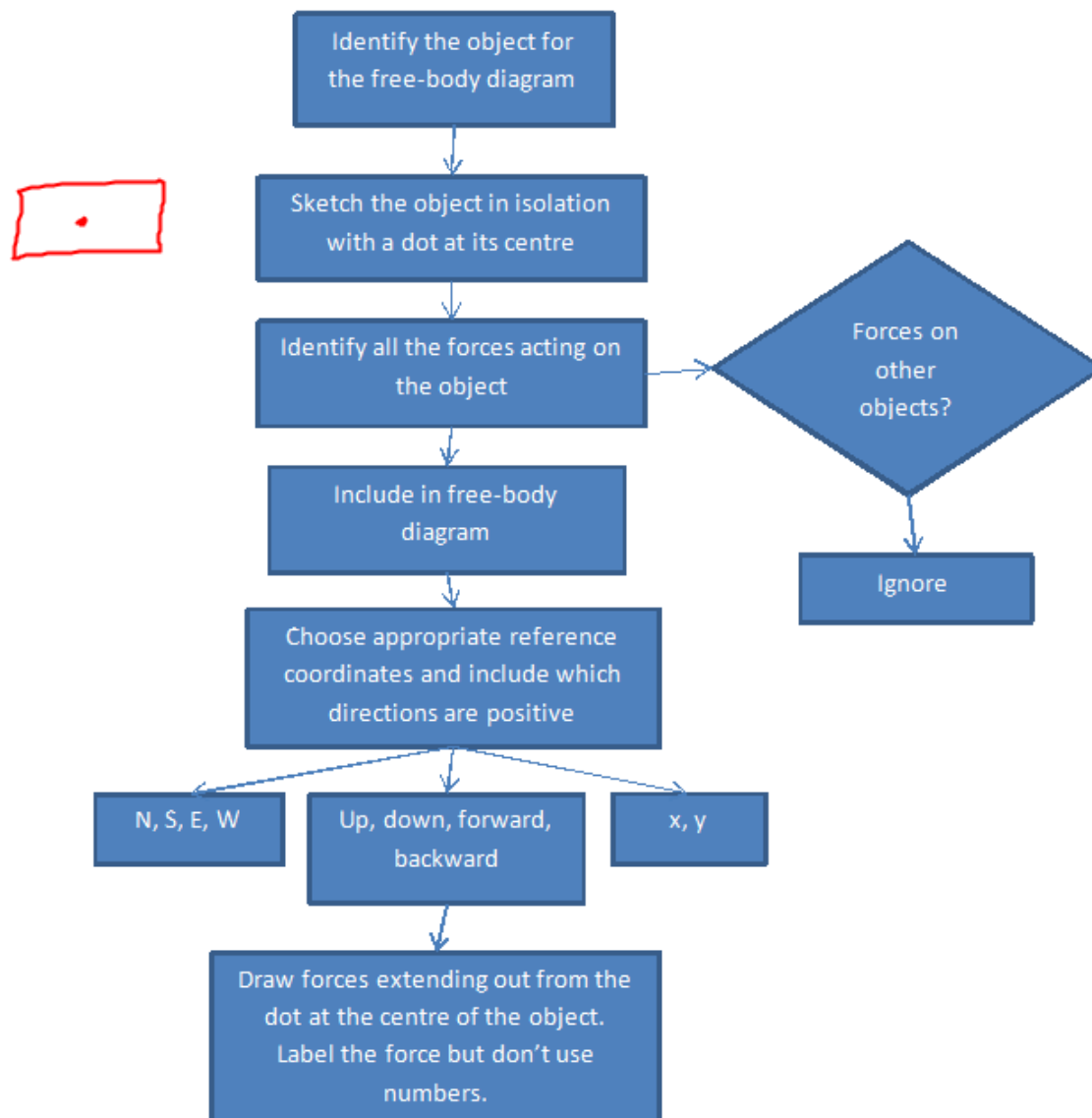
Free-Body Diagrams

Often, there are many forces acting on an object in any given situation and a system is needed to organize these forces. We will use something called a free-body diagram. A free-body diagram is a sketch that shows the object all by itself, isolated from all other objects.

When drawing a free-body diagram, it is important to denote your reference coordinates that are being used.



Flowchart for drawing free-body diagrams:



Ex. Sketch free-body diagram for a ball sitting on your palm.



We will see many types of forces acting on an object. Some common that you need to know include:

$$\vec{F}_g = \text{force due to gravity}$$

$$\vec{F}_a = \text{applied force}$$

$$\vec{F}_f = \text{force due to friction}$$

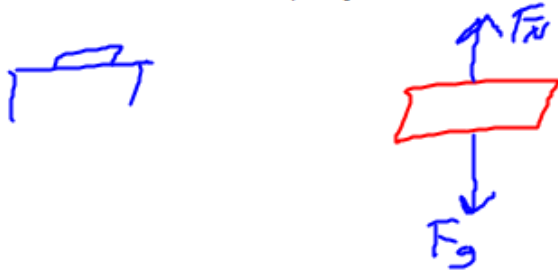
$$\vec{F}_T = \text{force of tension}$$

$$\vec{F}_N = \text{normal force}$$

$$\vec{F}_{NET} = \text{net force}$$

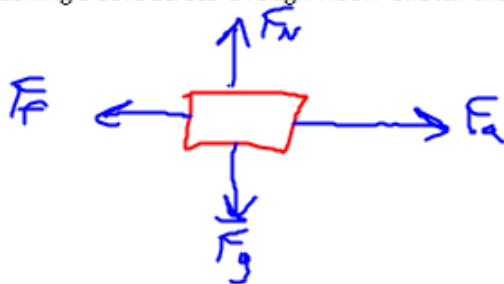
Each of the above forces will be discussed as they are introduced. When labeling your diagrams, you need to use the same symbols shown above. Capitals and lower case letters do matter.

Ex. Sketch a free-body diagram of a textbook sitting on a table.



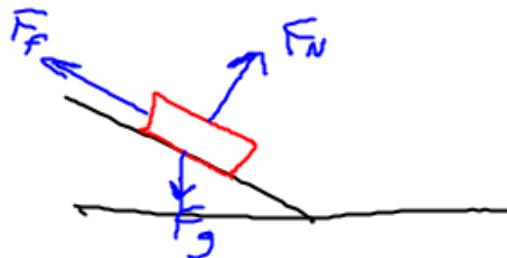
The normal force (\vec{F}_N) is force on an object that is perpendicular to the contact surface.

Ex. You are pushing a sofa across a rough floor. Sketch the free-body diagram of the sofa.



The force due to friction (\vec{F}_f) always opposes the motion of an object and is parallel to the contact surface.

Ex. A tobogganer is sliding down a hill. Sketch a free-body diagram of the toboggan.



Net Force

The net force acting on an object is the resultant of all forces acting on an object. Equilibrium exists if the net force is zero. We find it by finding the sum of all the forces acting on the object.

$$\vec{F}_{NET} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \dots$$

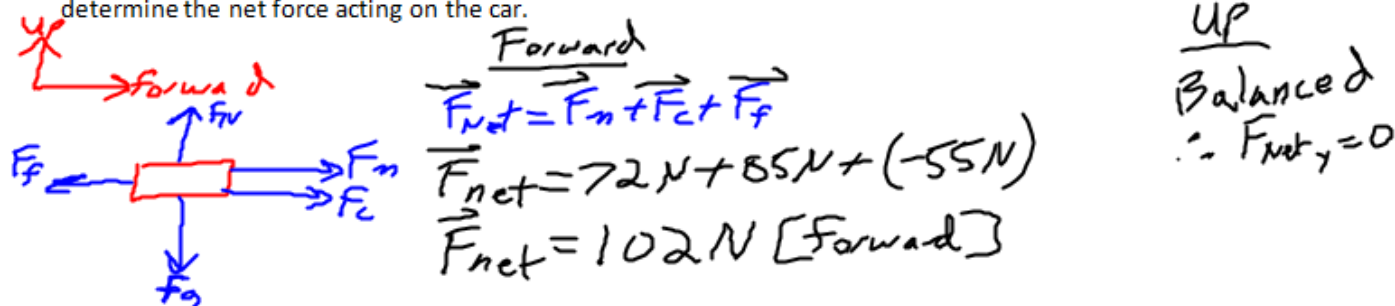
An object in equilibrium will move with a constant velocity.

An unbalanced force exists when the resultant of all the forces acting on an object does not equal zero. If no external unbalanced force acts on an object, its velocity will remain constant. If an unbalanced force exists, the object will begin to accelerate in that direction.

If the forces are parallel to one another, we can just add them together as positive and negative forces. If they are at an angle to one another, we have to add them as components of vectors.

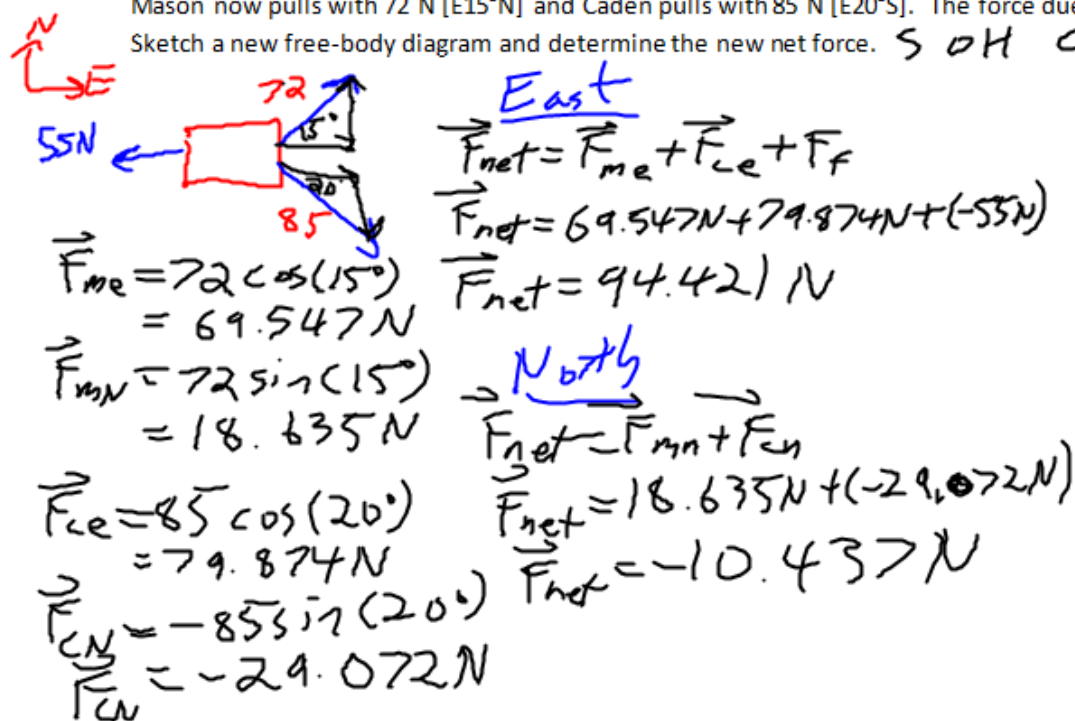
The unit of force is called the Newton in honour of Sir Isaac Newton. It is equivalent to the kg m/s^2 .

Ex. Mr. Birrell's Porsche is stuck in a snow drift. Two enterprising students attach two ropes to the Porsche and attempt to pull it out by pulling in the same direction. Mason pulls with a force of 72 N while Caden pulls with a force of 85 N. There is a force due to friction of 55 N acting on the car. Sketch a free-body diagram and determine the net force acting on the car.



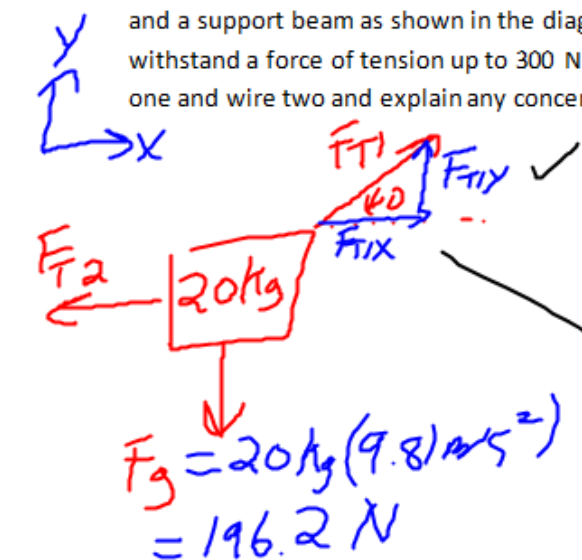
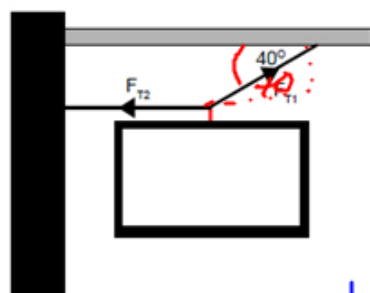
Ex. As Mason and Caden bravely pull the car, they notice a patch of ice on the road directly in front of them. To keep on pulling without wiping out on the ice, they must begin to pull at an angle as they walk around the ice.

Mason now pulls with 72 N [E15°N] and Caden pulls with 85 N [E20°S]. The force due to friction is still 55 N. Sketch a new free-body diagram and determine the new net force. SOH CAH TOA



Equilibrium $\rightarrow F_{NET} = 0$

Ex. A 20kg sign is supposed to hang from a pair of wires attached to the wall and a support beam as shown in the diagram. The wires that will be used can withstand a force of tension up to 300 N each. Determine the tension in wire one and wire two and explain any concerns you may have.



$$F_g = 20\text{kg}(9.8\text{m/s}^2) = 196.2\text{N}$$

Vertical

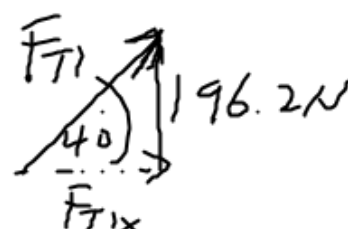
$$F_{NET} = F_g + F_{T1y}$$

$$0 = F_g + F_{T1y}$$

$$-F_g = F_{T1y}$$

$$-(-196.2\text{N}) = F_{T1y}$$

$$F_{T1y} = 196.2\text{N}$$



$$\sin 40^\circ = \frac{196.2\text{N}}{F_{T1}}$$

$$F_{T1} = 305.233\text{N}$$

$$\tan 40^\circ = \frac{196.2\text{N}}{F_{T1x}}$$

$$F_{T1x} = 233.822\text{N}$$

Horizontal

$$F_{NET} = F_{T2} + F_{T1x}$$

$$0 = F_{T2} + F_{T1x}$$

$$-F_{T1x} = F_{T2}$$

$$-233.822\text{N} = F_{T2}$$

$$F_{T1} = 300\text{N} [40^\circ \text{ above horizontal}]$$

$$F_{T2} = 200\text{N} [\text{Left}]$$

The force due to gravity is also called weight. It is defined as:

CONCERN: F_{T1} is over the limit for tension

$\vec{F}_g = m\vec{g}$	Where \vec{F}_g = Weight (Newtons, N, kg m/s ²) m = mass (kg) \vec{g} = acceleration due to gravity (m/s ²)
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Newton's First Law (Inertia)

"Every body continues in a state of rest or uniform velocity in a straight line, unless an external force acts on it."

"Every body..."

-anything with mass, anywhere

"...continues in a state of rest or uniform velocity in a straight line..."

-if sitting, will stay sitting

-if moving, will stay moving (straight line)

"...unless an external force acts on it."

- unless something pushes it
changing

Newton called the idea of an object resisting its state of motion inertia. This is why the First Law is sometimes called the law of inertia. We don't necessarily see it in daily life due to the effects of friction. The first law basically says:

$$\text{If } \vec{F}_{NET} = 0 \text{ then } \Delta \vec{v} = 0$$

Ex. Describe the motion of a hockey puck shot down the ice.

- keep moving in same direction at (almost) same speed until something hits it

Ex. Describe the motion of your binder sitting on your desk.

- it will sit there until you pick it up

Ex. Use Newton's First Law to explain why people are injured in head-on collisions in car accidents when they are not wearing their seatbelts.

- person has inertia; they will keep moving forward until something applies a force to them (steering wheel, dashboard, windshield, ground)

Newton essentially nicked his First Law from Galileo. Newton gets credit for it because he was the first to actually formally publish it and back it up with mathematical proofs.

Newton's Second Law (Motion)

The First Law says what will happen if there is no net force. The Second Law says what will happen when there is a net force present.

"When an external, unbalanced force acts on an object, the object will accelerate in the same direction as the force. The acceleration varies directly as the force, and inversely as the mass."

Break it down:

"When an external, unbalanced force..."

$F_{net} \neq 0$

"...accelerates in the same direction as the force..."

- velocity change is in direction of force

"The acceleration varies directly as the force..."

$\uparrow F \text{ then } \uparrow a$
 $\downarrow F \text{ then } \downarrow a$

"...and inversely as the mass."

$$F \propto m \downarrow a$$

$$F \propto m \uparrow a$$

From the above, we get the following mathematical relationship to express Newton's Second Law:

$\underline{\underline{\vec{F}_{NET} = m\vec{a}}}$	Where \vec{F}_{NET} = Net Force (Newtons, N, kg m/s ²) m = mass (kg) \vec{a} = acceleration (m/s ²)
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Ex. What is the net force acting on a 8.35 kg object if it is accelerated at 24 m/s²?

$$m = 8.35 \text{ kg}$$

$$\vec{a} = 24 \text{ m/s}^2$$

$$\vec{F}_{net} = ?$$

$$\vec{F}_{net} = m\vec{a}$$

$$\vec{F}_{net} = 8.35 \text{ kg}(24 \text{ m/s}^2)$$

$$\vec{F}_{net} = 200.4 \text{ N}$$

$$\boxed{\vec{F}_{net} \approx 2.0 \times 10^2 \text{ N}}$$

Ex. A lacrosse player exerts an average net horizontal force of 2.8 N [forward] on a 0.14 kg lacrosse ball while running with it in the net of his stick. Calculate the average horizontal acceleration of the ball while in contact with the lacrosse net.

$$m = 0.14 \text{ kg}$$

$$\vec{F}_{net} = 2.8 \text{ N}$$

$$\vec{a} = ?$$

$$\vec{F}_{net} = m\vec{a}$$

$$\vec{F}_{net} = \vec{a}m$$

$$\frac{2.8 \text{ N}}{0.14 \text{ kg}} = \vec{a}$$

$$\boxed{\vec{a} \approx 20. \text{ m/s}^2}$$

Ex. A person and an elevator have a combined mass of 532 kg. The elevator cable exerts a force of 625 N [up] on the elevator. Find the acceleration of the person.

$$\vec{F} = 625 \text{ N}$$

$$532 \text{ kg}$$

$$\vec{F}_{net} = m\vec{a}$$

$$\vec{F}_{net} = \vec{a}m$$

$$\vec{F}_T + \vec{F}_g = \vec{a}m$$

$$\vec{F}_g = m\vec{g}$$

$$\vec{F}_g = 532 \text{ kg}(9.8 \text{ m/s}^2)$$

$$\vec{F}_g = 5218.92 \text{ N}$$

$$\vec{a} = \frac{625 \text{ N} + (-5218.92 \text{ N})}{532 \text{ kg}}$$

$$\vec{a} = -8.63518797 \text{ m/s}^2$$

$$\boxed{\vec{a} = 8.64 \text{ m/s}^2 [\text{down}]}$$

Ex. Determine the acceleration of the elevator in the example above if the tension in the cable is only 320 N [up].

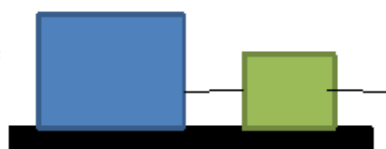
Ex. A 0.0500 kg piece of paper is dropped. While it falls there is a frictional force of 4.71×10^{-4} N. Determine the acceleration of the paper.

$F_f = 4.71 \times 10^{-4} \text{ N}$
 $m = 5.00 \times 10^{-5} \text{ kg}$
 $F_g = mg = 5.00 \times 10^{-5} \text{ kg} (9.81 \text{ m/s}^2) = 4.905 \times 10^{-4} \text{ N}$

$\vec{F}_{NET} = m\vec{a}$
 $\frac{\vec{F}_{NET}}{m} = \vec{a}$
 $\frac{F_f + F_g}{m} = \vec{a}$

$\vec{a} = \frac{4.71 \times 10^{-4} \text{ N} + (-4.905 \times 10^{-4} \text{ N})}{5.00 \times 10^{-5} \text{ kg}}$
 $\vec{a} = -0.39 \text{ m/s}^2$
 $\vec{a} = 0.390 \text{ m/s}^2 \text{ (down)}$

Ex. A 31 kg box and a 7.0 kg box are attached by a thin wire as shown in the picture. A person pulls on the wire attached to the box on the right. There is a force of friction of 243 N acting on the boxes. Find the force applied to the boxes if they accelerate to the right at 3.78 m/s^2 .



$\vec{F}_f = 243 \text{ N}$
 $m = 31 \text{ kg} + 7.0 \text{ kg} = 38 \text{ kg}$
 \vec{F}_g
 $\vec{a} = 3.78 \text{ m/s}^2$

$\vec{F}_{NET} = m\vec{a}$
 $F_f + F_a = m\vec{a}$
 $F_a = m\vec{a} - F_f$

$\vec{F}_a = 38 \text{ kg} (3.78 \text{ m/s}^2) - (-243 \text{ N})$
 $\vec{F}_a = 386.64 \text{ N}$
 $\vec{F}_a \approx 390 \text{ N [Right]}$

Pulley's change the direction of force. In any pulley problem we do, assume the rope and pulley are massless, have negligible width, and that the rope does not stretch or break. Often, one of the masses will be heavier than the other. This means the heavy mass will move down and the lighter mass move up.

Ex. Two masses are hanging from a pulley. If one mass is 15.25 kg and the mass two is 9.55 kg, determine the acceleration of each mass.

$F_{g1} = m_1 g$
 $F_{g1} = 15.25 \text{ kg} (9.81 \text{ m/s}^2)$
 $F_{g1} = 149.6025 \text{ N}$
 $F_{g2} = 9.55 \text{ kg} (9.81 \text{ m/s}^2) = 93.8855 \text{ N}$

$F_{NET} = m \vec{a}$
 $F_{g1} + F_{g2} = m \vec{a}$
 $\frac{F_{g1} + F_{g2}}{m} = \vec{a}$

$\vec{a} = \frac{149.6025 \text{ N} + (-93.8855 \text{ N})}{24.8 \text{ kg}}$
 $\vec{a} = 2.254717742 \text{ m/s}^2$

$m_1 = 2.25 \text{ m/s}^2 [\text{down}]$
 $m_2 = 2.25 \text{ m/s}^2 [\text{up}]$

Remember, a pulley can change the direction a force is acting in, but it does not change the magnitude of the force.

Newton's Third Law (Action-Reaction)

The third law deals with what happens when objects interact.

"For every action force there is an equal but opposite reaction force."

This means that when something applies a force, there will be an equal and opposite force back in the opposite direction. When you push on the handle of a lawnmower to make it go forward, it will push back against you in the opposite direction with just as much force. This is what you feel against your hand. It is important to remember that the two forces you are dealing with are each acting on different objects.

Contact forces are examples of action-reaction forces.

Ex. pushing a lawn mower
- normal force

Action: the tires on a car push the road

Reaction:

the road pushes on the tires

Action: In the lake, you push the water backwards:

Reaction:

the water pushes you forward

Action: A rocket pushes out exhaust

Reaction:

the exhaust pushes the rocket

Action: the earth pulls down on an apple

Reaction:

the apple pulls up on the earth

Ex. If the apple has a mass of 0.150 kg determine the acceleration of the Earth. ($M_e = 5.97 \times 10^{24}$ kg)

$$F_{\text{net}} = m_e \vec{a}_e$$

$$F_g = m_e a_e$$

$$m_a g = m_e a_e$$

$$\frac{m_a g}{m_e} = a_e$$

$$a_e = \frac{0.150 \text{ kg} (9.8) \text{ m/s}^2}{5.97 \times 10^{24} \text{ kg}}$$

$$a_e = 2.46482 \times 10^{-25} \text{ m/s}^2$$

Ex. When a rifle fires a bullet, the force the rifle exerts on the bullet is exactly the same (but in the opposite direction) as the force the bullet exerts on the rifle. This is what is commonly called "kick-back". If the bullet has a mass of 15 g and the rifle is 6.0 kg, the bullet leaves the 75 cm long rifle barrel moving at 70.0 m/s.

a) Find the acceleration of the bullet.

$$v_i = 0 \text{ m/s}$$

$$v_f^2 = v_i^2 + 2 \vec{a} \Delta d$$

$$\vec{a} = \frac{(70.0 \text{ m/s})^2}{2(0.75 \text{ m})}$$

$$v_f = 70.0 \text{ m/s}$$

$$\frac{v_f^2}{2 \Delta d} = a$$

$$\Delta d = 0.75 \text{ m}$$

$$a = ?$$

$$F_{\text{NET}} = m a$$

$$F_{\text{NET}} = 0.015 \text{ kg} (3266.667 \text{ m/s}^2) = 49 \text{ N}$$

b) Determine the force of the rifle on the bullet.

$$\vec{a} = 3266.667 \text{ m/s}^2$$
$$\vec{a} \approx 3.3 \times 10^3 \text{ m/s}^2$$

c) Determine the acceleration of the rifle.

$$F_b \approx 49\text{ N}$$

$$F_R \approx -49\text{ N}$$

$$\vec{F}_{\text{NET}} = m \vec{a}$$

$$\frac{\vec{F}_{\text{NET}}}{m} = \vec{a}$$

$$\vec{a} = \frac{-49\text{ N}}{6.0\text{ kg}}$$

$$\vec{a} \approx -8.2\text{ m/s}^2$$

d) Explain why the bullet accelerates more than the rifle if the forces are the same.

- the rifle has much more mass
 \rightarrow (inertia)

Friction

Friction acts to oppose motion. To find the direction of the force, look at the direction the object is travelling. The force due to friction acts 180° opposite.

It is not really known why friction acts the way it does. Some people believe it is the tiny imperfections in the two surfaces rubbing against each other. Others believe there are small electrostatic attractions between atoms of the two surfaces pulling on each other. Regardless of how friction acts on the microscopic scale, we can still predict what happens on the macroscopic scale.

Static friction is the friction that exists between two surfaces that are not moving relative to each other. Kinetic (sliding) friction is friction that exists between two surfaces that are moving relative to each other. In most cases, the static friction is greater than the kinetic friction.

If you push on a stationary object, you have to overcome the static friction. Once it is moving, it is often easier because you only have to overcome the smaller kinetic friction.

$$F_f = \mu F_N$$

Where F_f = Force due to friction

F_N = Normal force

μ = coefficient of friction

The coefficient of friction, μ , is different between any two unique surfaces. It has no unit and is an empirical discovery. This means the value of μ must be found by experimenting – there is no shortcut formula.

Each pair of surfaces also has two values of μ . The coefficient of static friction is μ_s and the coefficient of kinetic friction is μ_k . The smaller the value of μ , the less the force of friction there is between the two surfaces.

The static friction that you calculate is a measure of the maximum it can be. It can be any value between 0 and that maximum amount. The kinetic friction you calculate is the value of the friction – it is not a range.

$$F_{fs} \leq \mu_s F_n$$

$$F_{fk} = \mu_k F_n$$

Ex. A 11 kg piece of wood is sitting on top of another piece of wood. There is 43 N of maximum static friction between them. Find the coefficient of static friction between the two pieces of wood.

$$\mu_s = ?$$

$$F_{fs} = 43 \text{ N}$$

$$m = 11 \text{ kg}$$

$$F_{fs} = \mu_s F_N$$

$$F_{fs} = \mu_s (-mg)$$

$$\frac{F_{fs}}{-mg} = \mu_s$$

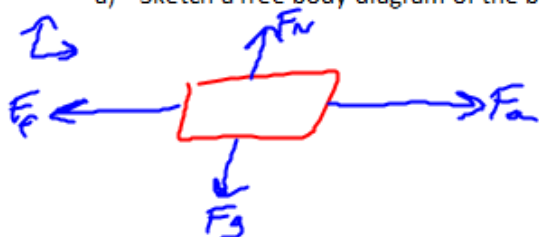
$$\mu_s = \frac{43 \text{ N}}{11 \text{ kg}(-9.8 \text{ m/s}^2)}$$

$$\mu_s = 0.40$$

As long as the surface is completely horizontal, we can say that $F_N = -F_g = -mg$.

Ex. A 15 kg steel box is sitting on a steel workbench. Wanting to assemble your new Malibu Stacy Dream Mansion, you try to push the box out of the way.

a) Sketch a free body diagram of the box.



b) You try to push against the box with a force of 25 N. Determine if anything will happen.

$$F_{fs} = ?$$

$$F_a = 25 \text{ N}$$

$$\mu_s = 0.74$$

$$F_{fs} = \mu_s F_N$$

$$F_{fs} = \mu_s (-mg)$$

$$F_{fs} = 0.74(-15 \text{ kg})(-9.8 \text{ m/s}^2)$$

$$F_{fs} = 108.84 \text{ N}$$

$$F_a < F_{fs}$$

\therefore Nothing happens
(Motion-wise)

c) Determine what happens when you push with a force of 73 N.

$$F_a < F_{fs}$$

\therefore Nothing happens

d) In anger, you Hulk out and push with a force of 120 N. What happens?

$$F_a = 120 \text{ N}$$

$$F_{fk} = ?$$

$$\mu_k = 0.57$$

$$m = 15 \text{ kg}$$

$$\vec{F}_{\text{net}} = m\vec{a}$$

$$\vec{F}_a + \vec{F}_{fk} = m\vec{a}$$

$$\frac{F_a + F_{fk}}{m} = \vec{a}$$

$$F_{fk} = \mu_k F_N = \mu_k (-mg)$$

$$= 0.57(-15)(-9.8)$$

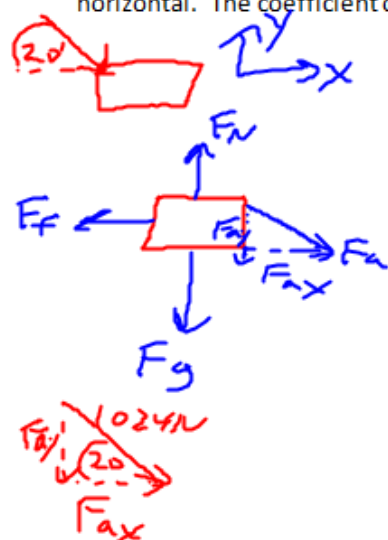
$$= 83.875 \text{ N}$$

$$\frac{120 \text{ N} + (-83.875 \text{ N})}{15 \text{ kg}} = \vec{a}$$

$$\vec{a} = 2.4 \text{ m/s}^2$$

By now, you have probably come to hate angles. This is a natural and healthy response, but one that must be overcome. In life, one rarely ever pushes or pulls at a perfectly horizontal angle. Life is not neat; it is a messy affair.

Ex. A person is pushing down on a 63 kg box so that the applied force is 1024 N at an angle of 20° above the horizontal. The coefficient of kinetic friction is 0.42. Determine the acceleration of the box.



$$\begin{aligned}\vec{F}_{ax} &= 1024 \text{ N} \cos 20^\circ \\ &= 962.25 \text{ N} \\ \vec{F}_{ay} &= 1024 \text{ N} (\sin 20^\circ) \\ &= -350.23 \text{ N}\end{aligned}$$

Vertical

$$\begin{aligned}F_{net} &= m\vec{a}_y \\ F_N + F_g + F_{ay} &= 0 \\ F_N + mg + F_{ay} &= 0 \\ F_N &= -mg - F_{ay} \\ \vec{F}_N &= -(63 \text{ kg})(-9.8 \text{ m/s}^2) - (-350.23 \text{ N}) \\ \vec{F}_N &= 968.26 \text{ N}\end{aligned}$$

Horizontal

$$\begin{aligned}F_{fk} &= \mu_k F_N \\ F_{fk} &= 0.42 (968.26 \text{ N}) \\ F_{fk} &= 406.6692 \text{ N} \\ \therefore F_{fk} &= -406.6692 \text{ N}\end{aligned}$$

Horizontal \vec{a}_x

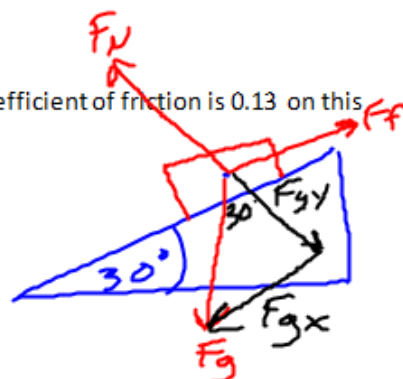
$$\begin{aligned}\vec{F}_{net} &= m\vec{a}_x \\ \frac{\vec{F}_{net}}{m} &= \vec{a}_x \\ \frac{\vec{F}_f + \vec{F}_{ax}}{m} &= \vec{a}_x \\ \vec{a}_x &= \frac{-406.6692 \text{ N} + 962.25 \text{ N}}{63 \text{ kg}} \\ \vec{a}_x &\approx 8.8 \text{ m/s}^2\end{aligned}$$

Inclined Planes

Running down an incline plane is easier than running horizontal. Downhill is faster. However, completing an incline plane problem in physics is neither quicker nor easier than a horizontal problem.

Ex. Determine the acceleration of a 15 kg box down a 30° slope if the coefficient of friction is 0.13 on this surface.

FBD:



Pick a convenient coordinate system:

Find F_g and break it down into parallel to the slope (F_{gx}) and perpendicular to the slope

(F_{gy}). F_{gy} will be equal to the normal force.

$$\begin{aligned} m &= 15 \text{ kg} \\ g &= 9.8 \text{ m/s}^2 \\ \vec{F}_g &= m\vec{g} \\ &= 15 \times 9.8 \\ &= 147.15 \text{ N} \end{aligned}$$

$$\begin{aligned} \vec{F}_{gy} &= -147.15 \text{ N} (\cos 30^\circ) \\ &= -127.44 \text{ N} \\ \vec{F}_{gx} &= +147.15 \text{ N} (\sin 30^\circ) \\ &= 73.58 \text{ N} \end{aligned}$$

$$F_N = -F_{gy} \quad (\text{balanced})$$

Find the force due to friction:

$$F_f = \mu F_N$$

$$F_{fk} = -16.5672 \text{ N}$$

$$F_{fk} = 0.13 (127.44 \text{ N})$$

Find the net force: (x direction)

$$\begin{aligned} \vec{F}_{netx} &= \vec{F}_{gx} + \vec{F}_f \\ \vec{F}_{netx} &= 73.58 \text{ N} + (-16.5672 \text{ N}) = 57.0128 \text{ N} \end{aligned}$$

Find the acceleration of the box:

$$\vec{F}_{netx} = m\vec{a}_x$$

$$\vec{F}_{netx} = \vec{a}_x$$

$$\vec{a}_x = \frac{57.0128 \text{ N}}{15 \text{ kg}}$$

$$\vec{a}_x = 3.8 \text{ m/s}^2 \quad [\text{downhill}]$$

Go over the example forty-two more times so that you can familiarize yourself with how to break the problem down into manageable chunks. Once you understand each part, it's all downhill from there...